JARAP

Journal of Mathematics & Statistics Science
www.jmss.jarap.org
Volume 1 Issue 1, 30 June 2018
PP. 8-10

ON A NON-TRIVIAL [15 8 3] PERFECT CODE DUE TO THE NON-ASSOCIATIVE PROPERTY OF (123)-AVOIDING CLASS OF AUNU PERMUTATION PATTERNS

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Key words

Perfect Codes
Hamming Bound
Extended codes
AUNU Patterns
Trivial and non-Trivial perfect codes
Dual code

Abstract

Here, a perfect [15 8 3] code is constructed from the [14 8 3] code earlier generated using the (u|u+v) construction method by the Authors. Let C be a linear code of length n. The code $\hat{C}$ of length $n+1$ obtained from C by adding one extra digit to each codeword in other to make each word in the new code have even weight is called the extended code of C. Sometimes an extended code results in a new code with improved error detection or error correction capabilities which are worth the price of a lower information rate. Our construction which adopts an earlier approach by the authors in the generation of a [14 8 3] code, is achieved by extending the generated code and further obtaining its generator and parity check matrices $\hat{G}\, and\, \hat{H}$ respectively. An established theorem by Hoffman et al is then used to ascertain its perfectness. The analysis of some other properties of perfect codes on this code is also examined.

1. INTRODUCTION

The application of the "Audu and Aminu"(AUNU for short)-avoiding permutation patterns in coding theory has been reported see [1], [2] and [3]. In [4], the existence of a generator matrix (in standard form) for codes from the cayley tables due to the non-associative (123)-avoiding patterns of AUNU numbers was studied. We say a code is perfect if any detected error in the code can be corrected. The class of perfect codes that exist are very useful. Unfortunately, there are few of linear perfect codes due the main problem in finding linear perfect codes that "the number $\binom{n}{0}+\binom{n}{1}+...+\binom{n}{t}$ must be a power of 2 (since $|C|$ is a power of 2). Since the class of perfect codes are useful and only a few exist, the study of this class of codes alongside their compliment such as duality is important. In this communication, we enumerate the construction of a [15 8 3]perfect code by extending the generated [14 8 3]in [3].

2. BASIC CONCEPTS

We consider without prove some important theorems by (Hoffman et al , 1992) which shall be useful in guarantying our findings. For the proof of these theorems, see [1 Hoffman]***Theorem 1*** A code of distance d will correct all error Patterns weight less than or equal to $\left(\frac{d-1}{2}\right)$. Moreover, there is at least one error pattern of weight $1+\left(\frac{d-1}{2}\right)$ which C will not correct.

Theorem 2 (The Hamming Bound) If C is a code of length n and distance $d = 2t+1\ or$

$2t+2$, then;

$|C| \le [\binom{n}{0}+\binom{n}{1}+...+\binom{n}{t}] \le 2^n \ or \ |C| \le \frac{2^n}{\binom{n}{0}+\binom{n}{1}+...+\binom{n}{t}}$

.**REMARK**

The Hamming bound is an upper bound for the number of code words in a code (linear or not) of length n and distance . $d = 2t+1$

Example 1

Compute an upper bound for the size M or dimension k of a linear code C with length $n = 6$ and distance $d = 3$.

Solution

From $d = 2t+1$,

We get $t = 1$. The Hamming bound gives

$$|C| \le \frac{2^n}{\binom{n}{0}+\binom{n}{1}+...+\binom{n}{t}}$$

$$\le \frac{2^6}{\binom{6}{0}+\binom{6}{1}} \text{ since } t=1$$

$$\le \frac{64}{1+6} = \frac{64}{7}$$

But $|C|$ must be a power of $2,$ so $|C| \le 8$ and thus $k \le 3$.

2.1 Perfect Codes

A code C of length n and odd distance $d = 2t+1$,is referred to as a perfect code if C attains the equality value in theorem 1, i.e If

$$|C| = \frac{2^n}{\binom{n}{0}+\binom{n}{1}+...+\binom{n}{t}}$$

Theorem 3

If C is a non-trivial perfect code of length n and distance $d = 2t+1$ then either

$$n = 23 \ and \ d = 7 \ or \ n = 2^r - 1 \ for \ some \ r \ge 2 \ and \ d = 3.$$

Proof

If a linear code of length n has distance $d = 2t+1$, then by theorem 1, C will correct all error Patterns of weight less than or equal to $t = \left(\frac{d-1}{2}\right)$. Thus every word of length n and weight less than or equal to t is a coset leader. There are precisely $\binom{n}{0}+\binom{n}{1}+ ... + \binom{n}{t}$ such words. But this is precisely the number of cosets if the code is perfect. We have thus proved the theorem. □

2.2 Extended Codes

Let C be an $[n \ k \ d]$ code over F_q, we define the extended code $\hat{C}$ to be the code $\hat{C} = \{x_1 x_2 x_3 ... x_{n+1} \in F_q^{n+1} \mid x_1 x_2 x_3 ... x_n \in C \ with \ x_1 + x_2 + x_3 + ... + x_{n+1} = 0\}$. $\hat{C} \, has \, \hat{d} = d \ or \ d+1.$

Let $G \ and \ H$ be the generator and parity check matrices respectively for C. Then the a generator matrix $\hat{G}$ for $\hat{C}$ can be obtain from G by adding an extra column to G so that the sum of the coordinates of each row of $\hat{G}$ is zero. $i.e \ \hat{G} = \left[\begin{array}{c|c} G & \begin{matrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_k \end{matrix} \end{array}\right]$.

A parity check matrix $\hat{H}$ for $\hat{C}$ is given by the matrix

$$\hat{H} = \left[\begin{array}{c|c} 1 \ 1 \ ... \ 1 & 1 \\ \hline & 0 \\ & . \\ H & . \\ & . \\ & 0 \end{array}\right]$$

3. METHODOLOGY

Here, we adopt the methods used by the authors in [3]. We recall using the (u|u+v) construction method in combining the [7 4 2] code generated due to AUNU patterns with the known [7 4 3] Hamming code to generate the [14 8 3] code. Consider the generator and parity check matrices of the generated [14 8 3] code and use them to obtain the generator and parity check matrices of its extended code. We thus proceed as follows. Now, the generator and parity check matrices G_O and H_O respectively as given in [3] are as follows;

$$G_O = \begin{bmatrix} 1&0&0&0&1&1&1&1&0&0&0&1&1&1 \\ 0&1&0&0&0&0&1&0&1&0&0&0&0&1 \\ 0&0&1&0&0&1&0&0&0&1&0&0&1&0 \\ 0&0&0&1&1&0&0&0&0&0&1&1&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&0&1&0&1&1&0 \\ 0&0&0&0&0&0&0&0&0&0&1&1&1&1 \end{bmatrix}$$

and

$$H_O = \begin{bmatrix} 1&0&0&1&1&0&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&1&0&0&0&0&0&0&0&0 \\ 1&1&0&0&0&0&1&0&0&0&0&0&0&0 \\ 1&0&0&0&0&1&1&0&1&1&1&1&0&0 \\ 0&1&0&0&1&0&1&1&0&1&1&0&1&0 \\ 0&0&1&0&1&1&0&1&1&0&1&0&0&1 \end{bmatrix}$$

From 2.2 above, clearly, we have;

$$\hat{G} = \left[\begin{array}{cccccccccccccc:c} 1&0&0&0&1&1&1&1&0&0&0&1&1&1&0 \\ 0&1&0&0&0&0&1&0&1&0&0&0&0&1&0 \\ 0&0&1&0&0&1&0&0&0&1&0&0&1&0&0 \\ 0&0&0&1&1&0&0&0&0&0&1&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&1&0&0&1&0&1&1 \\ 0&0&0&0&0&0&0&0&0&1&0&1&1&0&1 \\ 0&0&0&0&0&0&0&0&0&0&1&1&1&1&0 \end{array}\right]$$

and

$$\hat{H} = \left[\begin{array}{cccccccccccccc:c} 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ \hdashline 1&0&0&1&1&0&0&0&0&0&0&0&0&0&0 \\ 1&0&1&0&0&1&0&0&0&0&0&0&0&0&0 \\ 1&1&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&1&1&0&1&1&1&1&0&0&0 \\ 0&1&0&0&1&0&1&1&0&1&1&0&1&0&0 \\ 0&0&1&0&1&1&0&1&1&0&1&0&0&1&0 \end{array}\right]$$

The matrices $\hat{G}\, and\, \hat{H}$ are our desired generator and parity check matrices respectively. Clearly $\hat{G}$ is a generator matrix for a [15 8 3] code (the extended code). Next, using ***theorem 3***, we have $n = 2^r - 1 = 2^4 - 1 = 16 - 1 = 15$ satisfying $r \geq 2$.

$\therefore\ r = 4$. With $\hat{d} = d = 3$, we thus conclude from ***theorem 3*** that the [15 8 3] code is our desired Non-trivial Perfect code.

4. FINDINGS/RESULT

- The extended code of the [14 8 3] code in [3] is a non-trivial perfect code. As such, a more practical and applicable code has been deduced.
- The dual code $\hat{C}^{\perp}$ of $\hat{C}$ can be obtained using $\hat{H}$ as a generator matrix.

5. CONCLUSION/RECOMENDATION

A perfect single error correcting [15 8 3] code which would be of more practical importance has been constructed by extending the [14 8 3] code generated earlier by the authors. Further research on the dual code of the [15 8 3] code to determine its self duallity or otherwise is recommended.

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